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An Approach to Predicting the Threshold of Damage to an Angular Contact Bearing During Truncation

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A simple, approximate expression is developed, relating the mean stress with the fraction truncated at the contact between a ball and raceway of a loaded angular contact ball bearing. This expression is to serve as a rule-of-thumb for the bearing analyst to predict the likelihood of bearing damage due to truncation. The contact is said to be truncated when the contact area reaches beyond the edge of the raceway. We derive approximate expressions for the damage criteria due to excessive stress at the center of the contact ellipse and at the truncated edge of the contact. When either of these stresses exceeds the yield stress of the bearing materials, the damage threshold has been reached, and brinnelling will occur. Furthermore, we assert that when the fraction of the contact ellipse that is truncated exceeds 0.5, the ball will jam, and the bearing will be inoperable. These three criteria for successful operation beyond the initiation of truncation are combined into a single expression to be used for assessment of the danger of bearing damage. Our analysis provides for a first-order prediction of the conditions under which damage will result from a truncated contact.

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1. Introduction

When an angular contact bearing is subjected to an axial load, the contact between the ball and raceway is centered at some non-zero angle, the contact angle, from the centerline of the raceway, as shown in Figure 1. Calculation of the maximum stress within this contact was worked out years ago through mechanical analysis of the stresses and deflections that act on the bearing components,[1] allowing for the accurate prediction of the applied load at which damage will occur.[2] By definition, the static capacity of a ball bearing is the maximum static (stationary) load that can be supported without noticeable, subsequent effect on torque or vibration. A standard criterion for calculating static capacity is the calculated load required to produce a permanent indentation equal to 0.0001 times the ball diameter. Metallurgists utilize an indentation test to determine a compressive yield stress or an indentation yield stress. For 440C stainless steel at R_c 60, this value, the maximum allowable mean contact stress, is 340 ksi. We say that the static capacity of a bearing has been reached when the mean contact stress exceeds this value.

However, when the contact area and contact angle become so large that the contact exceeds the limits of the raceway (see Figure 1), the classic analytical methods no longer apply. Under these conditions, part of the contact is truncated, and the remaining contact area will change to equilibrate the modified forces.

The stress at the edge of the contact will increase rapidly with the fraction truncated, and the stress at the center of the contact will also increase as the total area of the contact is decreased. Therefore, the bearing materials will yield at a lower applied load than without truncation.

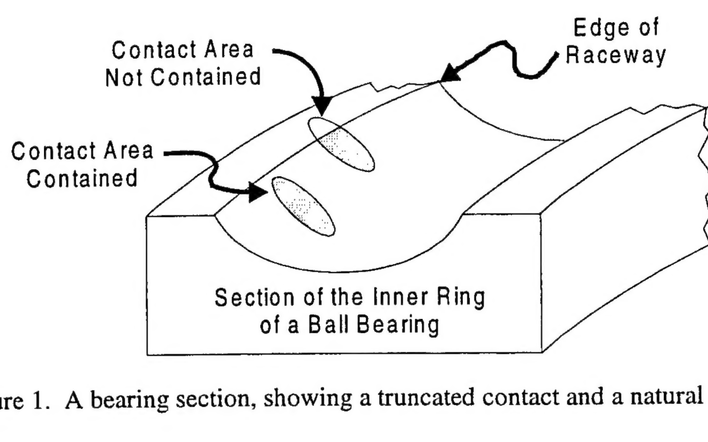

Figure 1. A bearing section, showing a truncated contact and a natural contact.

This situation is dangerous because it leads to bearing failure at a lower load than predicted without consideration of truncation. It is not difficult to predict the extent of truncation for a given bearing and a given set of operating conditions, and this is sometimes used as a criterion for rejecting a bearing design. However, the existence of truncation does not necessarily indicate that the bearing is unsuitable for a particular application. For example, the static capacity of a bearing may be larger than required by the loading situation, so that a small amount of truncation will not result in damage to the surface. In this case, a bearing may be approved for use in spite of truncation. Therefore, a quantitative approach for predicting the threshold of damage due to truncation of the contact in an angular contact bearing is needed to provide the bearing analyst with a tool for evaluating the static capacity of a contact beyond the truncation limit.

In this report, we present a pragmatic, engineering approach to predicting the threshold of damage due to a truncated contact. With the results presented below, a bearing analyst can take a calculated value of the mean contact stress and fraction truncated and predict the occurrence of brinnel dimples on the raceway surface. In this analysis, we determine approximate expressions for the maximum stress at the center of the truncated contact and at the truncated edge. When either of these stress values exceeds the compressive yield stress of the bearing materials, one may expect damage to occur. To simplify the application of this analysis, we write an expression that provides a safe margin for simultaneously satisfying the damage criteria from both the center and the edge of the contact.

2. Analysis

In this analysis, we will first determine the stress at the center and at the edge of a truncated ellipse, and then express these as functions of the mean stress under similar conditions without truncation and of the fraction truncated, f. The fraction truncated is defined as the length of the truncated portion along the major axis of the ellipse, divided by the total length of the major axis. For clarity, we will discuss the contact ellipse in its natural state (without truncation) and in the truncated state. To distinguish these states in the forthcoming analysis, the parameters that describe the truncated contact are marked with a prime. Figure 2 shows the stress distribution of the truncated contact. Inspection of this figure shows that the maximum stress may occur at one of two separate locations, the center or the edge, depending on the fraction truncated and the edge stress concentration factor, c. These two cases must be treated as independent criteria for the damage threshold. The stress at the center of the contact, S_0', and the stress at the truncated edge of the contact, S_e', are treated separately in Subsections 2.1 and 2.2, respectively.

2.1 Stress at the Center, S_0'

In this section, we express the stress at the center of the truncated contact ellipse, S_0', as a function of the mean stress in the natural contact ellipse, S_m, and the fraction truncated, f. The stress at the center of a truncated contact will be greater than that of a natural contact because a fraction of the surface area of the original ellipse is no longer bearing any of the load. Hertzian contact mechanics tells us that the maximum contact stress, neglecting any contribution at the center of the ellipse from the stress concentration at the edge, may be expressed as $S_0 = 3/2\ S_m$, where S_m is the mean stress. How-

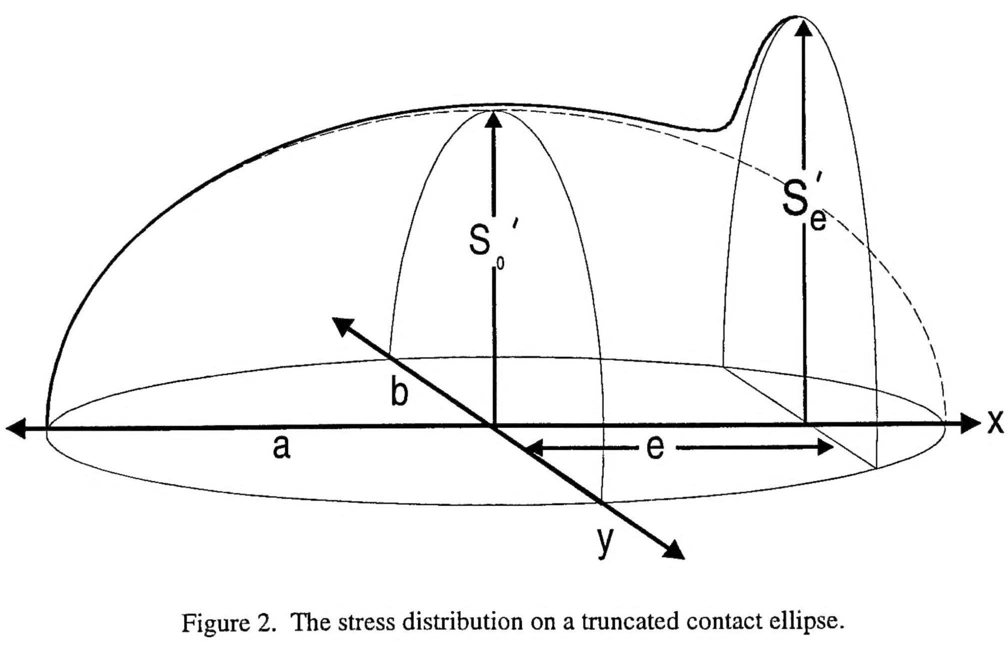

Figure 2. The stress distribution on a truncated contact ellipse.

ever, the mean stress in the remaining contact area will be increased from its natural state due to the loss of the contact area that has been truncated. As a result, the contact area and the mean stress in the natural state will be different from that after truncation. To clarify the forthcoming analysis, we define two coordinate systems: one for the natural contact ellipse, and one for the truncated ellipse (Figure 3).

Referring to Figure 3, the mean stress within the natural contact area is defined as the ratio of the total load, P, to the total area, A:

$$S_m = \frac{P}{A}. \tag{1}$$

Based upon the mechanics of an elliptical Hertzian contact,[1] we can express the mean stress of the complete contact ellipse in the primed coordinate system as:

$$S_m \propto P^{1/3}, \tag{2}$$

where the constant of proportionality depends on the elastic modulus, Poisson's ratio, and the radii of curvature of the ball and raceway.

When a fraction, f, of this ellipse is truncated, the contact area will change, but the total load must remain the same. We will neglect the edge effects, and treat the new contact area as a truncated ellipse. This new ellipse, also shown in Figure 3, will have a total area defined as A', equal to the sum of the truncated area A_T' and the remaining contact area A_R'. The total load necessary to form a complete contact ellipse of this size, P', is equal to the sum of the truncated load, P_T', and the remaining load, P_R'. The mean stress on the truncated contact ellipse may then be expressed as:

$$S_m' \propto P'^{1/3}, \tag{3}$$

where the constant of proportionality is equal to that of Eq. (2).

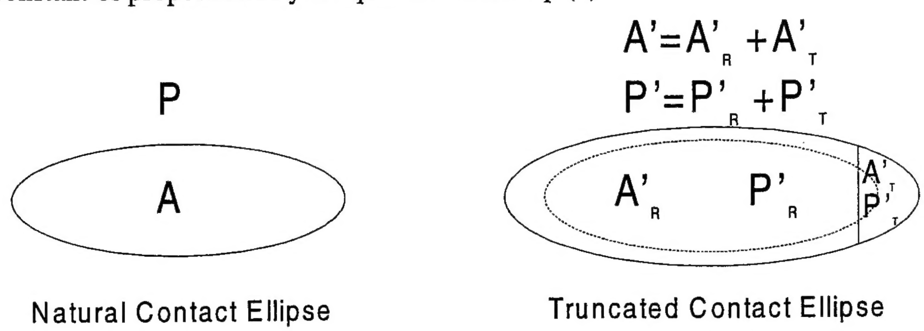

Figure 3. The contact ellipse before and after truncation. The dashed elipse oon the right side of figure represents the natural shape of the contact (without truncation).

Since the total load on the remaining contact area must equal the total load applied to the contact, P, we assert that $P_R' = P$, allowing us to rewrite Eq. (2) in terms of the truncated contact ellipse:

$$S_m \propto (P_R')^{1/3}. \tag{4}$$

Combining Eqs. (3)and (4), we find that :

$$S_m' = S_m \left(\frac{P'}{P_R'} \right)^{1/3}. \tag{5}$$

Finally, the stress at the center of the contact ellipse in the primed coordinate system may be written as:

$$S_0' = S_m \frac{3}{2} \left(\frac{P'}{P_R'} \right)^{1/3}. \tag{6}$$

The quantity contained within the parentheses, the ratio of the total load to the remaining load after truncation, is worked out in the appendix.

To find the various combinations of S_m and f that may lead to damage, we set the stress at the center equal to the compressive yield stress of the bearing materials, S_Y. The mean stress of the natural contact, normalized by the compressive yield stress of the bearing materials, is plotted against the fraction truncated in Figure 4. The line in this figure represents the damage threshold. For a given fraction truncated, when the ratio of the mean stress to the compressive yield stress exceeds this line, the center of the contact will be damaged.

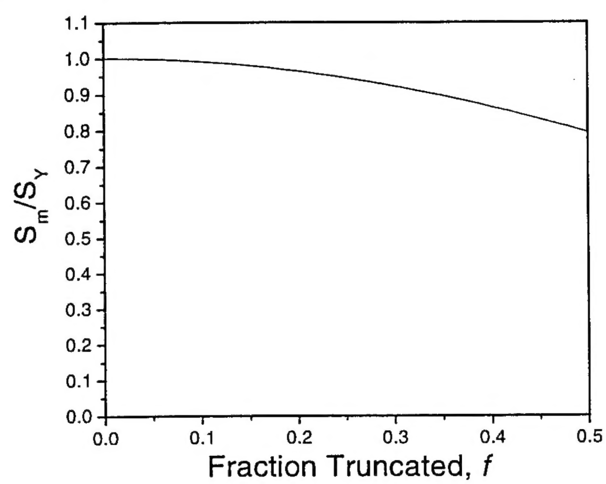

Figure 4. The threshold for damage at the center of a truncated ellipse.

2.2 Stress at the Edge, S_e'

In this section, we determine the stress at the edge of the truncated contact ellipse, and express it as a function of the mean stress of the natural contact ellipse and the fraction truncated. The contact stress at the edge of the raceway, the line of truncation, will be increased by a factor that is dependent upon the curvature at the edge; a sharper curve will result in a greater stress concentration and a greater amount of edge stress. Common practical values of the edge stress concentration factor, *c*, have been determined by Moyer and Neifert.[3] These values range from 1.7 to 1.8 for a typical blended corner radius. In the analysis presented below, we will estimate the edge stress by calculating the maximum stress of the natural stress ellipsoid on the line that would define the edge of the contact, then multiply this quantity by the edge stress concentration factor to determine the edge stress.

Referring to Figure 5, the surface of a semi-ellipsoidal pressure distribution is defined by:

$$1 = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{S_{x,y}}{S_0'}\right)^2 \tag{7}$$

Or, solving for the stress at arbitrary x and y position, $S'_{x,y}$,

$$S'_{x,y} = S_0'\left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2\right]^{1/2}, \tag{8}$$

and since the maximum stress always occurs at $y = 0$,

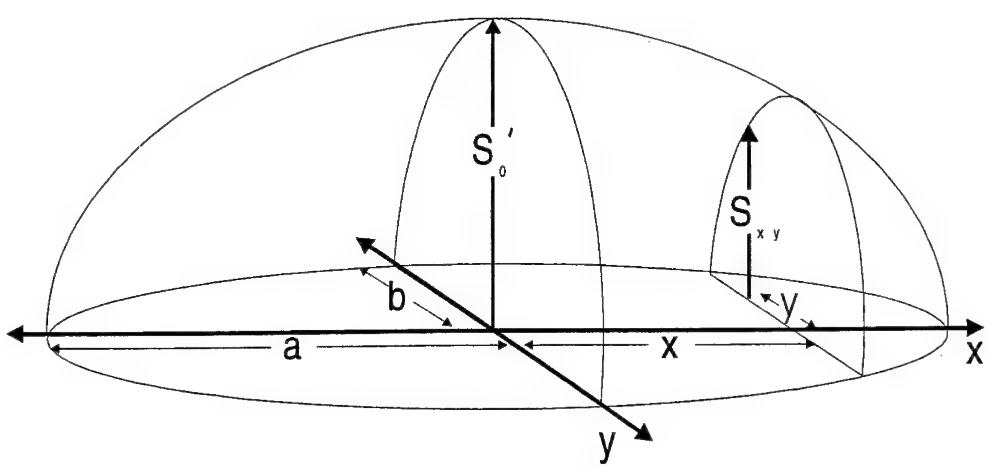

Figure 5. The stress distribution on an elliptical contact.

$$S'_x = S'_0\left[1-\left(\frac{x}{a}\right)^2\right]^{1/2} \tag{9}$$

From the quantities defined in Figure 5, we can express x in terms of the fraction truncated, f: $x = a(1-2f)$. Thus, Eq. (9) becomes:

$$S'_x = S'_0\left[1-(1-2f)^2\right]^{1/2}, \tag{10}$$

and the stress at the edge of the truncated contact is given as the product of the edge stress concentration factor, c, and the stress at S'_x:

$$S_e' = cS_x = \frac{3c}{2}S_m\left(\frac{P'}{P'_R}\right)^{1/3}\left[1-(1-2f)^2\right]^{1/2}. \tag{11}$$

Again, the parenthetical quantity is determined in the appendix.

To find the damage threshold at the edge, we set the edge stress equal to the compressive yield stress of the bearing materials. The mean stress, normalized by the compressive yield stress, is plotted in Figure 6. Here, we have chosen three representative values of the edge stress concentration factor to illustrate their effect on the static capacity during truncation. As stated above, c = 1.8 is a typical value for a well-designed and well-manufactured bearing.

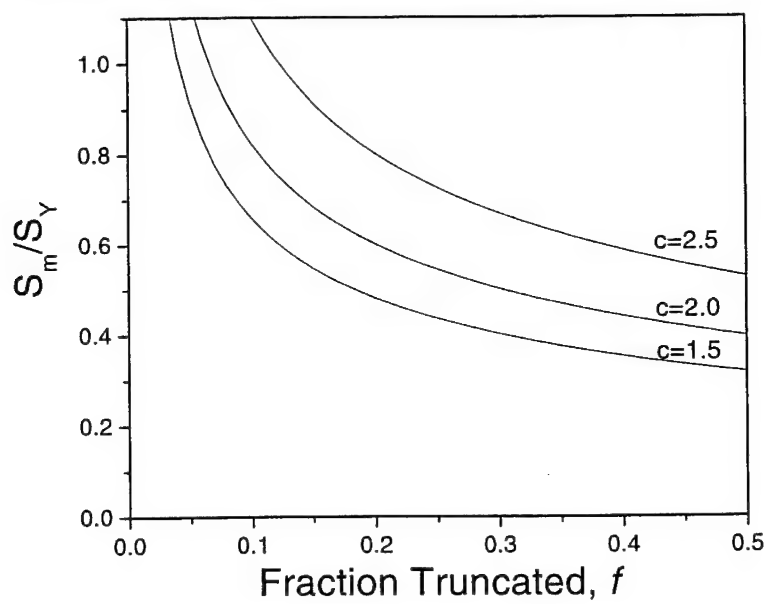

Figure 6. The threshold for damage at the edge of a truncated ellipse.

2.3 Approximate Criterion for Damage

In the preceding subsections, we determined the contact stress at the center of the contact ellipse and the center of the truncated edge of the contact. In addition to these criteria for the static capacity, we also stipulate that the fraction of truncation must not exceed 0.5. Truncation in excess of this amount will result in jamming the ball on the edge of the raceway, and the bearing will be inoperable. This effect is independent of the damage caused by truncation, and results only from the geometry of the bearing/raceway contact. In this subsection, we will use these three criteria to develop an approximate expression that simultaneously satisfies the damage threshold conditions at either of these two locations. This is done to provide the bearing analyst with a simple rule for predicting the likelihood of damage.

Figure 7 shows the threshold of damage, as determined by Eqs. (6) and (11). In this figure, the mean stress, normalized by the compressive yield stress, is plotted against the fraction truncated. Thus, when the contact stress exceeds either of the two lines, brinnel dimpling may be expected to occur. Finally, we reiterate that f may not be allowed to exceed 0.5.

The thick, straight line drawn diagonally across this figure represents the approximate damage threshold that we will use to simultaneously satisfy the three damage threshold criteria. Combinations of the normalized stress and fraction truncated that exceed this line may be in danger of damage due to excessive contact stress. Our combined truncation damage criterion may be expressed as:

$$\frac{S_m}{S_y} = 1 - 2f \tag{12}$$

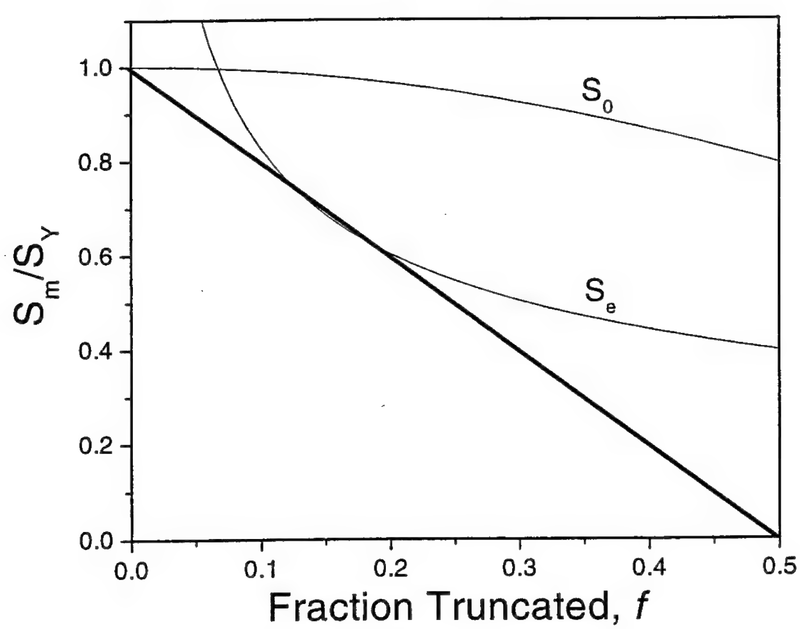

Figure 7. Summary of the three criteria for the damage threshold.

3. Conclusions

The static capacity of a truncated contact between a ball and raceway has been considered. We find that damage may occur at applied loads that are lower than those necessary to damage a natural contact. This damage may occur at the center of the contact or at the truncated edge of the ellipse. Approximate expressions for the contact stress as a function of the fraction truncated were derived as quantitative criteria for the damage threshold. As a third condition, we assert that the fraction truncated must not exceed 0.5, or the bearing will fail to operate. These three conditions were combined into a single expression, which can be used at a glance to predict the likelihood of bearing damage.

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Appendix

In this appendix, the ratio of the total load, P', to the remaining load on a truncated contact, P_R', is expressed as a function of the fraction truncated, f. Figure 1A shows the stress distribution of an elliptical contact.

The total load required to create an ellipse of this size is equal to the mean stress times the area, i.e., the area of the semi-ellipsoid shown in Figure 1A. The total load on the remaining area, after a fraction, f, had been truncated, is equal to the integral of the stress distribution function over the remaining area. The differential load carried by the volume located at (x,y), of width dx, is equal to the area of the semi-ellipse of major axis S_x, times dx:

$$dP_R = \frac{1}{2}(\pi S_x y)dx. \tag{1A}$$

Recall that for a plane ellipse,

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \tag{2A}$$

Substituting Eqs. (9) and (2A), into Eq. (1A):

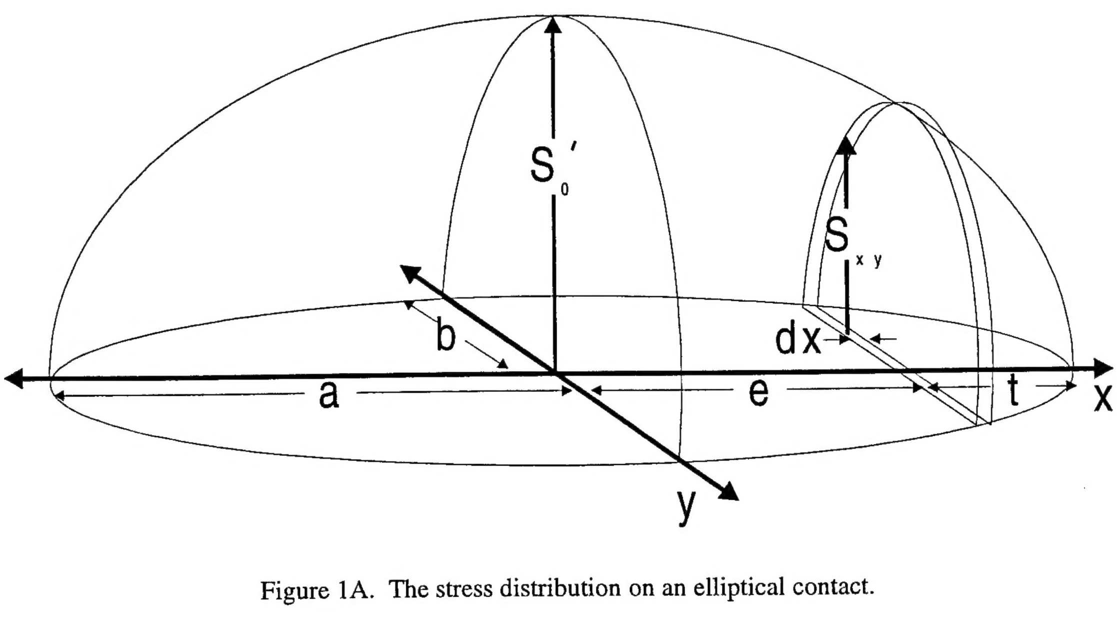

Figure 1A. The stress distribution on an elliptical contact.

$$dP_R' = \frac{\pi}{2} S_0' b \left[1 - \left(\frac{x}{a}\right)^2\right] dx \tag{3A}$$

Therefore, the total load on the area remaining after truncation to a value of x at the contact edge, e:

$$P_R' = \frac{1}{2} P' + \frac{\pi}{2} S_0' b \int_{x=0}^{x=e} \left[1 - \left(\frac{x}{a}\right)^2\right] dx \tag{4A}$$

$$= \frac{1}{2} P' + \frac{\pi}{2} S_0' b \left[e - \frac{e^3}{3a^2}\right]. \tag{5A}$$

Now we wish to define e, the point of truncation, in terms of the fraction truncated, f, defined as $\frac{t}{2a}$, where t is the length of the truncated region shown in Figure 1A. From this figure, we also note that since $t = a - e$,

$$e = a - 2af = a(1 - 2f) \tag{6A}$$

Substitution into Eq. (5A) yields,

$$P_R' = \frac{P'}{2} + \frac{\pi}{2} S_0' b \left[a(1 - 2f) - \frac{a^3 (1 - 2f)^3}{3a^2}\right] \tag{7A}$$

$$= \frac{P'}{2} + \frac{\pi}{2} S_0' ab \left[(1 - 2f) - \frac{(1 - 2f)^3}{3}\right] \tag{8A}$$

And since $S_0' = \frac{3}{2} S_m' = \frac{3}{2} \frac{P'}{\pi ab}$,

$$P_R' = \frac{P'}{2} \left\{1 + \frac{3}{2}\left[(1 - 2f) - \frac{1}{3}(1 - 2f)^3\right]\right\} \tag{9A}$$

Finally, the ratio of the total load to the remaining load is:

$$\frac{P'}{P_R'} = 2\left\{1 + \frac{3}{2}\left[(1 - 2f) - \frac{1}{3}(1 - 2f)^3\right]\right\}^{-1} \tag{10A}$$

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